Lesson: Origami and Mathematics

Learning Objectives:

- Discover origami and its roots in Japan and elsewhere
- Understand that origami can be a source of practical mathematics

Learning Outcomes:

- Know what origami is and fold some simple models
- Apply some mathematical concepts and problem solving to origami

Curriculum Links for Upper KS2 Mathematics:

Ensure that pupils **classify shapes** with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

Number: fractions (including decimals and percentages)

Y5: identify, name and write **equivalent fractions** of a given fraction, represented visually,

Y6: **multiply** simple pairs of proper **fractions**, writing the answer in its simplest form [for example, $(1/4) \times (1/2) = 1/2$]; **divide proper fractions** by whole numbers [for example, $(1/3) \div 2 = 1/6$]

Geometry: properties of shapes

Y5: know angles are measured in degrees: **estimate and compare acute, obtuse and reflex angles**

Y6: compare and **classify geometric shapes based on their properties** and sizes and **find unknown angles in** any triangles, quadrilaterals, and regular polygons

Keywords: Origami, Mathematics, Geometry, Fractions, Angles

Resources

Presentation: Origami and Mathematics (KS2 or KS3 version)

Materials: Teachers should prepare paper for each student - at least one square (approx. 10 to 20 cm) and two rectangles (e.g. A4 or A5). Double the amount of paper if they are to make more than one of each model.

Instructions: Paper Cup, Magazine Box, Seed Packet. Instructions are also provided within the PPT.

Additional Resources:

A visualiser is useful for demonstrating the folds to students.

Further Information and Resources :

- Learning Mathematics with Origami: www.atm.org.uk/Shop/Learning-Mathematics-with-Origami-book-and-pdf/act0101pk
- The Mathematics Behind the Folds: www.foldworks.net/home/articles/the-mathematics-behind-the-folds
- Geogebra origami animations: www.geogebra.org/u/tung+ken+lam

- British Origami Society: www.britishorigami.info
- Japan Origami Academic Society: https://origami.jp
- Nippon Origami Association: www.origami-noa.jp
  International Meetings on Origami in Science, Mathematics, and Education: http://osme.info

Introduction

- Use the presentation to introduce paper and paperfolding. Also show some examples of origami in Japan and elsewhere, today and in the past. Outline the links between origami and mathematics, science and engineering as appropriate to your class.

Estimated Time: 5-10 minutes

Task 1

- Students will make a Paper Cup (Angles and Lengths) and will need one square of paper.
- Before you start, show the students how to accurately and firmly fold the diagonal of a square (which is step 1) using a visualiser if you have access to one.
- Continue with the instructions to make the paper cup. Point out how mathematical vocabulary is used for clear instructions. NB: If the second fold is difficult, use the location point shown in extension activity 1.
- When finished, unfold the cup and return to the original square. Notice the crease lines in the paper: what are the angles that these lines make? Which segments have the same length? Use the standard mathematical symbols to show your findings.

Estimated Time: 10 minutes

Task 2

- Students will make a Magazine Box (Fractions) and will need one rectangle of paper
- Fold the 4 by 4 grid. Students should fold and unfold at each step for accuracy - if they fold and fold without unfolding, the thickness of the layers of paper will make the folds inaccurate.
- When the grid is complete, do some fraction work:
  - If the sheet is one unit, what fraction is one small rectangle? [one sixteenth]

- Fold the paper to show one quarter. Show a different way to make one quarter. How many sixteenths are there in one quarter? [four] What is one quarter divided by one sixteenth? [four]
- Show three quarters. Now halve the area. What is the resulting fraction? [three eighths]
- Show one eighth. What is one divided by one eighth? [8]
- Show one eighth. What is one half divided by one eighth? [4]
- What is three quarters divided by three sixteenths?
- What operations can you perform on these fractions, e.g. add, subtract, multiply and divide? Etc.

- Students can then make the magazine box. To do so they should:
  - Fold the short sides to the middle.
  - Fold the corners onto the crease lines that are one quarter along the longer edges.
  - Fold the inner flaps of paper outwards and over the corners.
  - Open the box and reinforce the creases of the edges to strengthen it.
- If you have time, ask students what happens if they use a rectangle that has different proportions?

Estimated Time: 15 minutes

Task 3

- Students will now make a Seed Packet (Angles and Lengths) and will need one rectangle of paper. Time depending, you could save this fold for another lesson and complete extension activity 1 or 2 instead.
- Follow the instructions and after each step, ask for the name of the shape, e.g. rectangle, irregular pentagon, irregular heptagon, etc.
- Also ask students to estimate some angles and if they are acute, right, reflex or obtuse angles.
- When finished, unfold and return to the original oblong. Notice the crease lines in the paper: what are the angles that these lines make? Which segments have the same length? Use the standard mathematical symbols to show your findings.

Estimated Time: 20 minutes

Extension Activities

1. The second fold of the paper cup has a precise location point: it is the intersection of the angle bisector of an acute-angled corner and the opposite edge. Why does this work? (See presentation for diagram)
2. For each model, make another one without seeing the instructions. Use the first model to help you remember the steps, or work out what is needed.
3. How can you divide an edge into thirds without using a ruler?
4. Make a grid that is 3 by 4. What fractions are possible?
5. Demonstrate or prove that the angles in the seed packet are multiples of 60°
6. Regular hexagons tessellate. What other shapes can you fold that also fit together without gaps or overlapping?

Further Activities for Students

- Find out about the origami theorems of Jun Maekawa and Jacques Justin about creases at a point.
- Search for engineering applications of origami.
- Find some origami books, websites and videos and make some more complicated origami.
- What other mathematics can be learned with origami?